On the nature of the Photon and the Quantum Vacuum

From Quantum Electrodynamics to Cosmology

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- The photon vector potential – energy (wave – particle) equation and the photon wave-function

The general solution of the vector potential issued from Maxwell’s equations yields that its amplitude $\alpha_0$ is proportional to the angular frequency $\omega$ so that for a $k$-mode electromagnetic field with wave vector module $|\vec{k}|$ we can write

$$\alpha_{0k} = \xi \omega_k$$

(1)

Based on the experimental evidence, the normalisation of the energy over a wavelength of a classical electromagnetic plane wave with circular polarization to Planck's expression of the quantized radiation energy $\hbar \omega_k$

$$E_k = \int 2\varepsilon_0 \omega_k^2 \alpha_{0k}^2 (\omega_k) |d^3r = \hbar \omega_k$$

(2)

entails that the constant $\xi$ has the value

$$|\xi| \propto \frac{1}{(2\pi)^{1/2}} \sqrt{\frac{\hbar}{8\alpha_{FS} \varepsilon_0 c^3}} = \frac{\hbar}{4\pi e c} \approx 1.747 \times 10^{-25} \text{ Volt m}^{-1} \text{s}^2$$

(3)

avec $\alpha_{FS} \approx 1/137$ the Fine Structure constant,

$\hbar = h/2\pi = 1.05457 \times 10^{-34} \text{ J s}$ Planck’s reduced constant,

$\varepsilon_0$ the vacuum electric permittivity $\varepsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$,

$e$ the electron charge $1.602 \times 10^{-19} \text{ Coulomb}$ and

$c$ the speed of light in vacuum $2.998 \times 10^8 \text{ m s}^{-1}$.

The fundamental physical properties characterizing both the particle (energy and momentum) and the electromagnetic wave (vector potential, wave vector and dispersion relation) nature of a single $k$-mode photon are related linearly to the angular frequency

$$\frac{E_k}{\hbar} = |\vec{p}_k| = \frac{\alpha_{0k}}{|\xi|} = |\vec{k}| \omega_k$$

(4)
The vector potential \( \mathbf{A}_{k\lambda}(\mathbf{r}, t) \) for a \( k \)-mode photon with the quantized amplitude \( \xi \omega_k \) writes in the plane wave representation with polarization \( \lambda \to 2 \) (Right or Left circular):

\[
\mathbf{A}_{k\lambda}(\mathbf{r}, t) = \omega_k \left[ \xi \mathbf{\hat{e}}_z e^{i(k\mathbf{r} - \omega_k t + \phi)} + cc \right] = \omega_k \mathbf{\hat{u}}_{k\lambda}(\mathbf{r}, t)
\]

(5)

Of course, \( \mathbf{A}_{k\lambda}(\mathbf{r}, t) \) satisfies the wave propagation equation

\[
\nabla^2 \mathbf{A}_{k\lambda}(\mathbf{r}, t) - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{A}_{k\lambda}(\mathbf{r}, t) = 0
\]

(6)

\( \mathbf{A}_{k\lambda}(\mathbf{r}, t) \) also satisfies Schrödinger’s equation for the energy with the massless particle Hamiltonian \( \mathbf{H} = -i\hbar \nabla \mathbf{V} \) and eigenvalue \( \hbar \omega_k \),

\[
i \hbar \frac{\partial}{\partial t} \mathbf{A}_{k\lambda}(\mathbf{r}, t) = \mathbf{H} \mathbf{A}_{k\lambda}(\mathbf{r}, t)
\]

(7)

and finally \( \mathbf{A}_{k\lambda}(\mathbf{r}, t) \) satisfies a linear time-dependent equation for the vector potential amplitude operator \( \widetilde{\mathbf{\alpha}}_0 = -i\xi \mathbf{\hat{e}}_z \mathbf{\hat{V}} \) with eigenvalue \( \xi \omega_k \),

\[
i \frac{\xi}{\hbar} \frac{\partial}{\partial t} \mathbf{A}_{k\lambda}(\mathbf{r}, t) = \widetilde{\mathbf{\alpha}}_0 \mathbf{A}_{k\lambda}(\mathbf{r}, t)
\]

(8)

which is the photon vector potential equation.

✓ The coupling of equations (7) and (8) yields the photon vector potential – energy (wave – particle) equation,

\[
i \left( \frac{\xi}{\hbar} \right) \frac{\partial}{\partial t} \mathbf{A}_{k\lambda}(\mathbf{r}, t) = \begin{pmatrix} \widetilde{\mathbf{\alpha}}_0 \\ \mathbf{H} \end{pmatrix} \mathbf{A}_{k\lambda}(\mathbf{r}, t)
\]

(9)

✓ Consequently, the vector potential \( \mathbf{A}_{k\lambda}(\mathbf{r}, t) \) with the quantized amplitude \( \alpha_{0k} = \xi \omega_k \) may play the role of a real wave function for the photon in a non-local representation that can be suitably normalized.
Thus, the probability $P_k(\vec{r})$ for detecting a $k$-mode photon around the point $\vec{r}$ is proportional to the square of the angular frequency

$$P_k(\vec{r}) \propto |\tilde{a}_{k\ell}(\vec{r},t)|^2 \propto \xi^2 \omega_k^2 \quad (10)$$

We also deduce the amplitude $|\tilde{E}_{k\ell}|$ of the electric field for a single $k$-mode photon

$$|\tilde{E}_{k\ell}| = \left| -\frac{\partial}{\partial t} \tilde{a}_{k\ell}(\vec{r},t) \right| \propto \xi \omega_k^2 \quad (11)$$

which is also proportional to the square of the angular frequency.

Therein, it is of fundamental importance to investigate experimentally the dependence of the electric field of a free photon on its angular frequency.

- **Photon spatial extension**

In Quantum Electrodynamics (QED) theory photons are assumed to be point particles. This hypothesis permits to use a convenient mathematical formalism in order to study the interaction of light with matter. However, many experiments have shown that photons are not really point particles and that they are spatially extended over a wavelength. Obviously, the real nature of individual photons seems to be more complicated than the simple point particle representation and we deal with this aspect in the following.

The energy density equivalence between the classical and the quantum mechanical formulations for an electromagnetic field reduced to a single photon state writes

$$2\varepsilon_0 a^2_\omega a^2_{0k} (\omega_k) = \frac{\hbar \omega_k}{V} \quad (12)$$

The normalization of the energy over a wavelength of a classical electromagnetic plane wave with circular polarization, issued from Maxwell's equations, to Planck's expression of the quantized radiation energy is given by equation (2).
Comparison of relations (2) and (12) yields the definition of the quantized volume $V_k$ of the $k$-mode photon which appears to be an intrinsic property depending on $\omega_k^3$ (that is the cube of the wavelength)

$$V_k = \left( \frac{\hbar}{2\varepsilon_0 \varepsilon^2} \right) \omega_k^{-3}$$

(13)

The dependence on $\omega_k^{-3}$ is in agreement with the density of states theory as well as the experimental evidence according to which a photon cannot be localized in a volume less than the cube of its wavelength $\lambda^3$. The energy density of a single photon is

$$W_k = 2\varepsilon_0 \varepsilon^2 \omega_k^4$$

(14)

The dependence on $\omega_k^4$ is analogue to that of the energy density of a radiating dipole.

Hence, we can say that the energy and the momentum of the photon are not carried by a point particle and can be obtained using the quantization volume $V_k$ and the quantized vector potential amplitude:

$$E_k = \int_{V_k} 2\varepsilon_0 \alpha_{0k}^2 \omega_k d^3r = \int_{V_k} 2\varepsilon_0 \varepsilon^2 \omega_k^4 d^3r = 2\varepsilon_0 \varepsilon^2 \omega_k^4 V_k = \hbar \omega_k$$

(15)

Considering a circular polarization the momentum writes:

$$\vec{P}_k = \int_{V_k} \varepsilon_0 \vec{e}_{k\lambda} \times \vec{\beta}_{k\lambda} d^3r = \varepsilon_0 \left( \sqrt{2} \omega_k \alpha_{0k} \left( \frac{1}{c} \sqrt{2} \omega_k \alpha_{0k} \right) V_k \frac{\vec{k}}{|\vec{k}|} \right) = \hbar \vec{k}$$

(16)

where $\vec{e}_{k\lambda}$ and $\vec{\beta}_{k\lambda}$ denote the electric and magnetic fields of a single mode $k$. 
The fact that the energy and momentum of the photon can be expressed through $V_k$ also supports that the photon is not in reality a point but has a minimum spatial extension corresponding to the quantization volume in which the quantized vector potential oscillates over a period. A photon with wavelength $\lambda_c$ can be detected within a volume of the order of the cube of its wavelength.

✓ Consequently instead of “wave-particle” we should more properly employ the term “wave-corpuscle”.

It is worth noting that according to the relation (13) for micro-waves and radio-wave frequencies the spatial extension of a single photon corresponds to macroscopic dimensions. In microwave cavities the simplest mode $k$ of the electromagnetic field cannot subsist within a volume smaller than that defined by the cut-off wavelengths in different spatial directions.

✓ The QED formalism might evolve towards a 3-dimensional representation i.e., creation and annihilation operators of quantized three dimensional photons $a^+_{\nu_-}$ and $a_{\nu_+}$ respectively.

What is a Photon like?

The experimental evidence shows that the photon is neither a point particle nor a continuous wave. Furthermore, no experiments have ever shown that a single photon state is a harmonic oscillator. A photon with angular frequency $\omega_k$ seems to be the minimum “quantum” of the electromagnetic field, composed of a quantized vector potential with amplitude $|\xi|^2 \omega_k$ oscillating over a period $2\pi/\omega_k$, with Left or Right circular polarization, whose energy integrated over a quantized volume $V_k$ extended along the wavelength equals $\hbar \omega_k$. Thus, it should be more appropriate to consider the photon as an indivisible “wave-corpuscle”, extended over a wavelength, composing the electromagnetic field and propagating along the vector potential wave function. The quantized vector potential gives rise to local oscillating electric and magnetic fields over a wavelength which also propagate along the wave function.
• **Ground level of the electromagnetic field, a quantum vacuum component**

According to the relations (4), for \( \omega_k \rightarrow 0 (\lambda_k \rightarrow \infty) \) all the physical properties of the photon vanish. However, even when \( \omega_k = 0 \), that is in complete absence of energy and vector potential, the field \( \Xi_{k,\lambda} \) expressed in (5) does not cancel and reduces to the vacuum field \( \Xi_{0,\lambda} \), involving the polarization, which can be described as a vector field as well as a quantum mechanical operator

\[
\Xi_{0,\lambda} = \xi \hat{E}_\lambda e^{i\phi} + \xi^* \hat{E}_\lambda^* e^{-i\phi}
\]  \hspace{1cm} (17)

\[
\Xi_{0,\lambda} = \xi a_{k,\lambda} \hat{E}_\lambda e^{i\phi} + \xi^* a_{k,\lambda}^* \hat{E}_\lambda^* e^{-i\phi}
\]  \hspace{1cm} (18)

the presence of the creation operator means that the vacuum field \( \Xi_{0,\lambda} \) is a dynamic entity capable of inducing electronic transitions.

Equations (17) and (18) represent the ground state of the radiation field \( (\omega_k = 0, \forall k) \), in complete absence of energy and vector potential and can thus be assimilated to the vacuum state. Obviously, the field \( \Xi_{0,\lambda} \) is a real component of the vacuum, having the physical units Volt m\(^{-1}\) s\(^2\) implying an electric potential nature of the quantum vacuum.

- The vacuum field \( \Xi_{0,\lambda} \) is the generating function for photons. Photons are vibrations of the vacuum field, extended over a wavelength and propagating along the vector potential wave function. Hence, the energy (and consequently the mass) appears to be the direct result of real vacuum field vibrations.

- Furthermore, the vacuum field \( \Xi_{0,\lambda} \) which is present at any coordinate in space involves the photon polarizations entailing that the entanglement behaviour may lay directly in the vacuum nature.

- It is also important remarking that the electron/positron charge is expressed naturally through the fine structure constant \( \alpha_{FS} \) and the vacuum constants \( \xi \) and \( \mu_0 \).

\[
|e| = (4\pi)^2 \alpha_{FS} \frac{\xi}{\mu_0}
\]  \hspace{1cm} (19)

where \( \mu_0 \) is the vacuum magnetic permeability \( \mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1} \).

The physical meaning of the last relation is that the elementary charges (of matter and antimatter) might also be issued from the quantum vacuum like photons.
It can be shown that the field $\Xi_{0,\omega}$ is capable of interacting with the electrons in matter since it is expressed as a function of photon creation and annihilation operators $a_{k,\lambda}^\dagger$ and $a_{k,\lambda}$ respectively. An interaction Hamiltonian between the vacuum and the electrons can be defined in order to describe the spontaneous emission effect. $\Xi_{0,\omega}$ can also be used in Bethe’s calculation for the Lamb shift getting identical results.

Finally, the Casimir effect can also be interpreted as a result of the vacuum radiation pressure “seen” by the electrons in their frame due to their periodic motion in the vacuum field.

Following the relation (3) the electric potential character of the vacuum expressed through the constant $\xi$ entails that every charge moving in space with an acceleration $\vec{\gamma}$ will experience an electric potential $U_{\text{vacuum}}$ due precisely to the field $\Xi_{0,\omega}$ whose amplitude is

$$U_{\text{vacuum}} = |\xi \vec{\gamma}|$$

(20)

This could play an important role in the cosmic vacuum energy. Also, fluctuations of the vacuum field $\Xi_{0,\omega}$ due to high charge and mass concentrations in space my give rise to low frequency electromagnetic fields contributing to the cosmic electromagnetic background.

✓ Consequently, the properties and behaviour of the vacuum field $\Xi_{0,\omega}$ may have a quite significant contribution to the cosmological constant $\Lambda$.

Finally, the vacuum is not a sea of photons with all frequencies and polarizations with infinite energy leading to the well-known Quantum Electrodynamics singularity, which is in contradiction with the astrophysical observations entailing the quantum vacuum catastrophe characterization, but it is composed of an electric potential field capable of generating photons and elementary charges.

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