The nature of the Photon and the Quantum Vacuum

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- **The photon *vector potential* equation**

The general solution of the vector potential issued from Maxwell's equations yields that its amplitude α_0 is proportional to the angular frequency ω so that for a *k*-mode electromagnetic field we can write

$$\alpha_{0k} = \xi\,\omega_k \tag{1}$$

Based on the experimental evidence, the normalisation of the energy over a wavelength of a classical electromagnetic plane wave with circular polarization to Planck's expression of the quantized radiation energy $\hbar\,\omega_k$

$$\int 2\varepsilon_0 \omega_k^2 \alpha_{0k}^2(\omega_k) d^3 r = \hbar\omega_k \tag{2}$$

entails that the constant ξ has the value

$$|\xi| \propto \left| \frac{1}{(2\pi)^{3/2}} \sqrt{\frac{\hbar}{8\alpha_{FS}\varepsilon_0 c^3}} \right| = \left| \frac{\hbar}{4\pi e c} \right| \approx 1.747\ 10^{-25}\ Volt\ m^{-1} s^2 \tag{3}$$

avec $\alpha_{FS} = 1/137$ the Fine Structure constant, $\hbar = h/2\pi$ = 1.05457 10^{-34} J s Planck's reduced constant, ε_0 the vacuum electric permittivity $\varepsilon_0 = 8.85\times10^{-12}\,\mathrm{F\,m^{-1}}$, *e* the electron unit charge 1.602 10^{-19} Coulomb and *c* the speed of light in vacuum 2.998 10^{8} m s^{-1}.

The fundamental physical properties characterizing both the particle (energy and momentum) and the wave (vector potential, wave vector and dispersion relation) nature of a single *k*-mode photon are related linearly to the angular frequency

$$\frac{E_k}{\hbar} = \frac{|\vec{p}_k|}{\hbar/c} = \frac{\alpha_{0k}}{\xi} = |\vec{k}|c = \omega_k \tag{4}$$

The vector potential $\vec{\alpha}_{k\lambda}(\vec{r},t)$ for a *k*-mode photon with the quantized amplitude $\xi\omega_k$ writes in the plane wave representation with polarization λ = 1,2 (Right or Left circular):

$$\vec{\alpha}_{k\lambda}(\vec{r},t) = \omega_k \left[\xi\,\hat{\varepsilon}_\lambda\, e^{i\left(\vec{k}\cdot\vec{r} - \omega_k t + \theta\right)} + cc \right] = \omega_k\, \vec{\Xi}_{k\lambda}(\vec{r},t) \tag{5}$$

Of course, $\vec{\alpha}_{k\lambda}(\vec{r},t)$ satisfies the wave propagation equation

$$\vec{\nabla}^2\vec{\alpha}_{k\lambda}(\vec{r},t)-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{\alpha}_{k\lambda}(\vec{r},t)=0 \qquad (6)$$

$\vec{\alpha}_{k\lambda}(\vec{r},t)$ also satisfies Schrödinger's equation for the energy with the massless particle Hamiltonian $\tilde{H}=-i\hbar c\vec{\nabla}$ and eigenvalue $\hbar\omega_k$,

$$i\hbar\frac{\partial}{\partial t}\vec{\alpha}_{k\lambda}(\vec{r},t)=\tilde{H}\,\vec{\alpha}_{k\lambda}(\vec{r},t) \qquad (7)$$

and finally $\vec{\alpha}_{k\lambda}(\vec{r},t)$ satisfies a linear time-dependent equation for the vector potential amplitude operator $\tilde{\alpha}_0=-i\xi c\vec{\nabla}$ with eigenvalue $\xi\,\omega_k$,

$$i\,\xi\frac{\partial}{\partial t}\vec{\alpha}_{k\lambda}(\vec{r},t)=\tilde{\alpha}_0\,\vec{\alpha}_{k\lambda}(\vec{r},t) \qquad (8)$$

which is the *photon vector potential equation*.

The coupling of both last equations yields the photon *vector potential – energy* (wave –particle) equation,

$$i\begin{pmatrix}\xi\\ \hbar\end{pmatrix}\frac{\partial}{\partial t}\vec{\alpha}_{k\lambda}(\vec{r},t)=\begin{pmatrix}\tilde{\alpha}_0\\ H\end{pmatrix}\vec{\alpha}_{k\lambda}(\vec{r},t) \qquad (9)$$

Consequently, the vector potential $\vec{\alpha}_{k\lambda}(\vec{r},t)$ with the quantized amplitude may play the role of a real wave function for the photon in a non-local representation that can be suitably normalized.

Thus, the probability $P_k(\vec{r})$ for detecting a *k*-mode photon around the point $\vec{r}$ is proportional to the square of the angular frequency

$$P_k(\vec{r})\propto\left|\vec{\alpha}_{k\lambda}(\vec{r},t)\right|^2\propto\xi^2\omega_k^2 \qquad (10)$$

We also deduce the amplitude $\left|\vec{\varepsilon}_{k\lambda}\right|$ of the electric field for a single *k*-mode photon

$$\left|\vec{\varepsilon}_{k\lambda}\right|=\left|-\frac{\partial}{\partial t}\vec{\alpha}_{k\lambda}(\vec{r},t)\right|\propto\xi\omega_k^2 \qquad (11)$$

which is also proportional to the square of the angular frequency.

In Quantum Electrodynamics (QED) theory photons are assumed to be point particles. This hypothesis permits to use a convenient mathematical formalism in order to study the interaction of light with matter. However, many experiments have shown that photons are not really point particles and that they are spatially extended over a wavelength. Obviously, the real nature of individual photons seems to be more complicated than the simple point particle representation and we deal with this aspect in the following.

- **Photon spatial extension**

The normalization of the energy over a wavelength of a classical electromagnetic plane wave with circular polarization, issued from Maxwell's equations, to Planck's expression of the quantized radiation energy

$$\int 2\varepsilon_0 \omega_k^2 \alpha_{0k}^2 (\omega_k) d^3 r = \hbar\omega_k \qquad (12)$$

results to the definition of the quantized volume V_k of the *k*-mode photon which is an intrinsic property

$$V_k = \left(\frac{\hbar}{2\varepsilon_0 \xi^2} \right) \omega_k^{-3} \qquad (13)$$

The energy and the momentum of the photon are note carried by a point particle and can be obtained using the quantization volume V_k and the quantized vector potential amplitude:

$$E_k = \int_{V_k} 2\varepsilon_0 \alpha_{0k}{}^2 \omega_k^2 d^3 r = \int_{V_k} 2\varepsilon_0 \xi^2 \omega_k^4 d^3 r = 2\varepsilon_0 \xi^2 \omega_k^4 V_k = \hbar\omega_k \qquad (14)$$

The energy density of a single photon is

$$W_k = 2\varepsilon_0 \xi^2 \omega_k^4 \qquad (15)$$

Considering a circular polarization the momentum writes:

$$\vec{P}_k = \int_{V_k} \varepsilon_0 \vec{\varepsilon}_{k\lambda} \times \vec{\beta}_{k\lambda} d^3 r = \varepsilon_0 \left(\sqrt{2}\omega_k \alpha_{0k}\right)\left(\frac{1}{c}\sqrt{2}\omega_k \alpha_{0k}\right) V_k \frac{\vec{k}}{\left|\vec{k}\right|} = \hbar \vec{k} \qquad (16)$$

where $\vec{\varepsilon}_{k\lambda}$ and $\vec{\beta}_{k\lambda}$ denote the electric and magnetic fields of a single mode *k*.

The fact that the energy and momentum of the photon can be expressed through V_k supports that the photon is not in reality a point but has a minimum spatial extension corresponding to the quantization volume in which the quantized vector potential oscillates over a period. A photon with wavelength λ_κ can be detected within a volume of the order of the cube of its wavelength λ_κ^3.

Consequently instead of "*wave-particle"* we should more properly employ the term "***wave-corpuscle***".

It is worth noting that according to the relation (13) for micro-waves and radio-wave frequencies the spatial extension of a single photon corresponds to macroscopic dimensions.

The QED formalism should evolve towards a 3-dimensional representation i.e., creation and annihilation operators of quantized three dimensional photons $a^{+}_{V_k\lambda}$ and $a_{V_k\lambda}$ respectively.

What is a Photon like?

The experimental evidence shows that ***the photon is neither a point particle nor a continuous wave***. A photon with angular frequency ω_k seems to be the minimum "quantum" or "segment" of the electromagnetic field, composed of a quantized vector potential with amplitude $\xi\omega_k$ oscillating over a period $2\pi/\omega_k$, with Left or Right circular polarization, whose energy integrated over a quantized volume V_k extended along the wavelength equals $\hbar\omega_k$. Thus, it should be more appropriate to consider the photon as an indivisible "***wave-corpuscle***", extended over a wavelength, composing the electromagnetic field and propagating along the vector potential wave function. This gives rise to local oscillating electric and magnetic fields which also propagate successively along the wave function.

- **Ground level of the electromagnetic field, a quantum vacuum component**

According to the relations (4), for $\omega_k \to 0$ all the physical properties of the photon vanish. However, even when $\omega_k = 0$, that is in complete absence of energy and vector potential, the field $\Xi_{k\lambda}$ expressed in (5) does not cancel and reduces to the vacuum field $\Xi_{0\lambda}$, involving the polarization, which can be described as a vector field as well as a quantum mechanical operator

$$\vec{\Xi}_{0\lambda} = \xi \hat{\varepsilon}_\lambda e^{i\varphi} + \xi^* \hat{\varepsilon}_\lambda^* e^{-i\varphi} \qquad (17)$$

$$\tilde{\Xi}_{0\lambda} = \xi a_{k\lambda} \hat{\varepsilon}_\lambda e^{i\varphi} + \xi^* a_{k\lambda}^+ \hat{\varepsilon}_\lambda^* e^{-i\varphi} \qquad (18)$$

the presence of the creation operator means that the vacuum field $\tilde{\Xi}_{0\lambda}$ is a dynamic entity capable of inducing electronic transitions.

Equations (17) and (18) represent the ground state of the radiation field $(\omega_k = 0, \forall k)$, in complete absence of energy and vector potential and can thus be assimilated to the vacuum state. Obviously, the field $\Xi_{0\lambda}$ is a real component of the vacuum, having the physical units Volt m^{-1} s^{2} implying an electric potential nature of the quantum vacuum.

The vacuum field $\Xi_{0\lambda}$ is the generating function for photons. Photons are vibrations of the vacuum field, extended over a wavelength and propagating along the vector potential wave function. Hence, the energy and mass appear to be the direct result of real vacuum field vibrations.

It is also important remarking that the electron charge can be expressed naturally through the fine structure constant $\alpha_{FS} = 1/137$ and the vacuum constants ξ and μ_0.

$$e = (4\pi)^2 \alpha_{FS} \frac{\xi}{\mu_0} \qquad (19)$$

where μ_0 is the vacuum magnetic permeability $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$.

The physical meaning of the last relation is that the elementary charges might also be issued from the quantum vacuum like photons.

It can be shown that the field $\Xi_{0\lambda}$ is capable of interacting with the electrons in matter since it is expressed as a function of photon creation and annihilation operators $a_{k\lambda}^+$ and $a_{k\lambda}$ respectively. An interaction Hamiltonian between the vacuum and the electrons can be defined in order to describe the spontaneous emission effect. $\Xi_{0\lambda}$ can also be used in Bethe's calculation for the Lamb shift getting identical results. Finally, the Casimir effect can also be interpreted as a result of the vacuum radiation pressure "seen" by the electrons in their frame due to their periodic motion in the vacuum field.

Following the relation (3) the electric potential character of the vacuum expressed through the constant ξ entails that every charge moving in space with an acceleration $\vec{\gamma}$ will experience an electric potential U_{vacuum} due precisely to the field $\Xi_{0\lambda}$

$$U_{vacuum} = \xi \left| \vec{\gamma} \right| \tag{20}$$

This could play an important role in the cosmic vacuum energy and consequently might contribute significantly to the cosmological constant.

Consequently, the vacuum is not a sea of photons with all frequencies and polarizations with infinite energy as described in QED, which is in contradiction with the astrophysical observations leading to the *quantum vacuum catastrophe*, but it is composed of an electric potential field capable of generating photons and elementary charges.

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